

## FINAL EXAMINATION

*This examination is open-book, open-notes. Confidentiality is required during examinations. Please strictly observe academic integrity. Examinations should be your own personal work. During examinations, other people (classmates, friends, professors --- except Troy and Prof. Starr) are CLOSED; do not discuss examination materials until after the exam has been collected.* You may use any published sources (cite them). If you have questions, e-mail them to Prof. Starr at [rstarr@ucsd.edu](mailto:rstarr@ucsd.edu). Notation not specified here is taken from Starr's *Introduction*.

**Answer any five (5) of the six questions below. Papers with six questions answered will be evaluated on the lowest scoring five. Enjoy!!**

**Question 1 (example 1) and question 2, below, deal with a typical weighted voting plan:** Consider a voting plan for a group of voters to choose the best one of ten possibilities: A, B, C, D, E, F, G, H, I, J. Each voter submits a ballot ranking the possibilities. The voting procedure then gives his first place choice a weight of 10; the second place choice is given a weight of 9; ...; the tenth place choice is given a weight of 1. For each possibility, the weighted votes of all the voters are then added up. The possibility achieving the highest total of weighted votes is declared the winner. A tie-breaking rule may be needed.

**1.** Answer the questions in the following two examples.

**Example 1:** Using the weighted voting plan above, let there be three voters with the following rankings. Topmost proposition is weighted 10, bottom is weighted 1: Larry            Moe            Curly

A	D	G
B	E	H
C	F	I
D	G	J
E	H	A
F	I	B
G	J	C
H	A	D
I	B	E
J	C	F

**(question 1, example 1, continues next page)**

Given this ranking G gets 21 points and looks like a winner (Prof. Starr can't do all these sums in his head --- he thinks that's right). Can Moe restate his preferences to make D a winner? How?

**Example 2:** Consider majority voting over pairwise alternatives. The preferences of three voters (ranked top to bottom) over three possibilities are

<u>Larry</u>	<u>Moe</u>	<u>Curly</u>
A	B	C
B	C	A
C	A	B

The procedure is that the chairman picks two possibilities and conducts a majority vote on those two. Then there is a majority vote of the third remaining possibility versus the winner of the first vote (For example: Vote on A versus B; A wins. Then vote A versus C.).

Show that the chairman can arrange for any one of A, B, or C to be chosen, by selecting the order of voting.

2. Evaluate the weighted voting procedure (described above question 1 and implemented in question 1, example 1) in terms of the Sen version of the Arrow axioms. Does the procedure fulfill: Pareto Principle? Independence of Irrelevant Alternatives? Non-Dictatorship? Unrestricted Domain? Explain.

3. Recall in Starr's *Introduction*, that in defining household demand behavior we used the truncated budget set (where the length of the consumption vector is limited to a maximum value of  $c$ ),

$$\tilde{B}^i(p) = \{x \mid x \in \mathbb{R}^N, p \cdot x \leq \tilde{M}^i(p)\} \cap \{x \mid \|x\| \leq c\}.$$

We defined demand behavior as

$$\tilde{D}^i(p) = \{x \mid x \in \tilde{B}^i(p) \cap X^i, u^i(x) \geq u^i(y) \text{ for all } y \in \tilde{B}^i(p) \cap X^i\}.$$

We then established in Theorem 5.2, under additional assumptions, that  $\tilde{D}^i(p)$  is well defined (non-empty).

(i) Show that this result depends on the truncation of  $\tilde{B}^i(p)$ . That is, define  $B^i(p) = \{x \mid x \in \mathbb{R}^N, p \cdot x \leq \tilde{M}^i(p)\}$  and

$D^i(p) = \{x \mid x \in B^i(p) \cap X^i, u^i(x) \geq u^i(y) \text{ for all } y \in B^i(p) \cap X^i\}$ . Note that these functions  $D$  and  $B$  are not restricted to  $\{x \mid \|x\| \leq c\}$  (these functions do not have  $\sim$  superscript).

**(question 3, part i, continues next page)**

Show that for some utility functions,  $u^i$  and prices (where  $p_k=0$  for some goods  $k$ ),  $D^i(p)$  may not be well defined under the same situation where  $\tilde{D}^i(p)$  will be well-defined. (Hint: You may find the following example useful. Let  $i$ 's preferences be described by the utility function  $u(x_1, x_2) = (x_1+1)(x_2+1)$ , let the budget be  $= 100$ ,  $p=(p_1, p_2)=(0,1)$ .)

(ii) Explain why we should find the restriction to  $\{ x \mid \|x\| \leq c \}$  undesirable as part of the description of the household opportunity set. Is it helpful that the restriction is not binding in equilibrium (Lemma 7.1, Theorem 11.1)? Explain.

**4.** The usual U-shaped cost curve model of undergraduate intermediate economics includes a small non-convexity (diminishing marginal cost at low output levels). This is a violation of our usual convexity assumptions on production (P.I or P.V). Consider the general equilibrium of an economy displaying U-shaped cost curves. It is possible that a general equilibrium exists despite the small violation of convexity. After all, P.I and P.V are sufficient, not necessary, conditions. If a general equilibrium does exist despite the small nonconvexity, will the allocation be Pareto efficient? Does the First Fundamental Theorem of Welfare Economics apply? Explain.

**5.** Explain the significance of the following results. That is, do not merely restate the theorems; explain why economists should be interested in them. Explain why they are helpful in doing pure and applied economic analysis.

(a) The Existence of General Equilibrium, Theorems 1.2 , 7.1, and 11.1 of Starr's *Introduction*.

(b) The First Fundamental Theorem of Welfare Economics, Theorem 12.1 of Starr's *Introduction*.

**6.** Consider a general equilibrium model with two commodities, that is  $N=2$ . Denote the two goods  $x$  and  $y$ . The possible consumption space for each household is the nonnegative quadrant, that is  $X^h = \mathbb{R}_+^2$  for all  $h \in H$ , (fulfilling C.I, C.II, C.III). Let all households  $h \in H$ , have the same utility function :

$$u^h(x, y) = \sqrt{x} + \sqrt{y} + [x]$$

where  $[x]$  denotes the greatest integer  $\leq x$ .  $u^h$  is discontinuous at many points in  $\mathbb{R}_+^2$  (discontinuous in the neighborhood of integer values of  $x$ ). Endowments are strictly positive in  $x$  and  $y$  but left unspecified.

[Hint: It may help to think about demand behavior in the neighborhood of  $(p_x, p_y) = (1/2, 1/2)$  with household income equal to  $0.5 = 1/2$ . As the price vector moves from  $(.51, .49)$  to  $(.50, .50)$ , note the change in the demand for  $x$ . The optimizing consumption vector moves (discontinuously) from (approximately)  $(.5, .5)$  to  $(1, 0)$ .

In addition, note that we corrected --- in class --- the assumptions of Theorem 12.1; sufficient conditions are C.II (unbounded possible consumption set) and C.IV (weak monotonicity); C.V (continuity) is not required. ]

**(a)** Assume the usual other assumptions of the general equilibrium model (continuity and convexity of production technologies, adequacy of income, and so forth). Assume that Walras' Law is fulfilled. Make and state clearly any other assumptions you need (consistent with the setting above). Can we be sure that there is a competitive equilibrium in this economy? Explain fully why or why not.

**(b)** When there exists a competitive equilibrium, will the equilibrium allocation be Pareto efficient? Explain fully why or why not.